RÉVISIONS PHYSIQUE

Traitement du signal

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Physique Traitement du signal 1

Équation différentielle linéaire ⟹ principe de superposition.
⟹ invariant temporellement
↳ ne dépend pas de l'instant initial

Système stable ⟺ coef. de même signe
(dans l'équa. diff. d'ordre 2 homogène)

Entrée	Sortie
Impulsion	Nulle
Échelon	Continu
1-périodique sinusoïdal	1-périodique sinusoïdal
1-périodique quelconque	1-périodique quelconque

↳ pour un système linéaire ↳ forme différente, en général

Représentation complexe → pour un signal sinusoïdal

$$e(t) \longrightarrow \underline{e}(t) = E e^{j\omega t}$$
$$\frac{d}{dt} e(t) \longrightarrow j\omega\, \underline{e}(t)$$
$$\int e(t)\, dt \longrightarrow \frac{1}{j\omega}\, \underline{e}(t)$$

$\underline{e}$ (Entrée) → Fonction de transfert $\underline{H}$ → $\underline{S} = \underline{H} \cdot \underline{e}$ (Sortie)

Gain $G = |\underline{H}|$; Phase $\varphi = \mathrm{Arg}(\underline{H})$. $G_{dB} = 20 \log G$

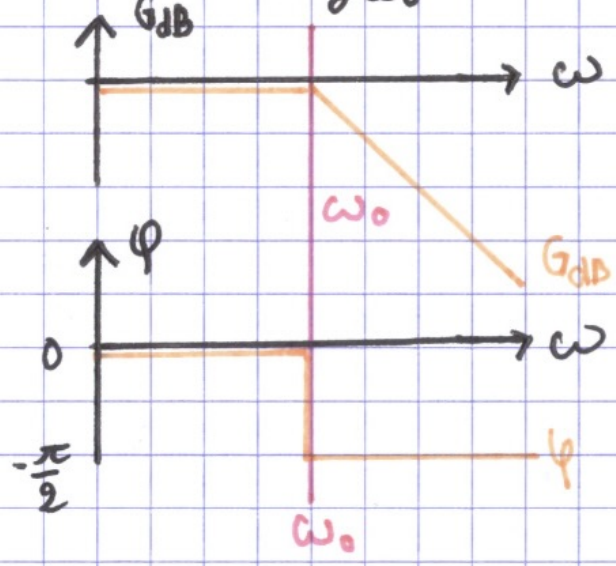
Diagramme de Bode : graphe de G_{dB} et φ en échelle lin-log

Filtres usuels - 1er et 2nd ordre

PREMIER ORDRE

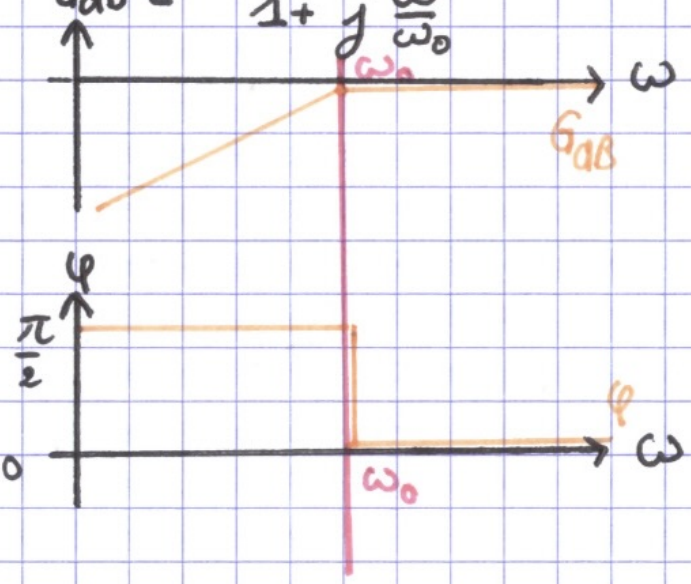
Passe-bas

$$\underline{H} = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$



Passe haut

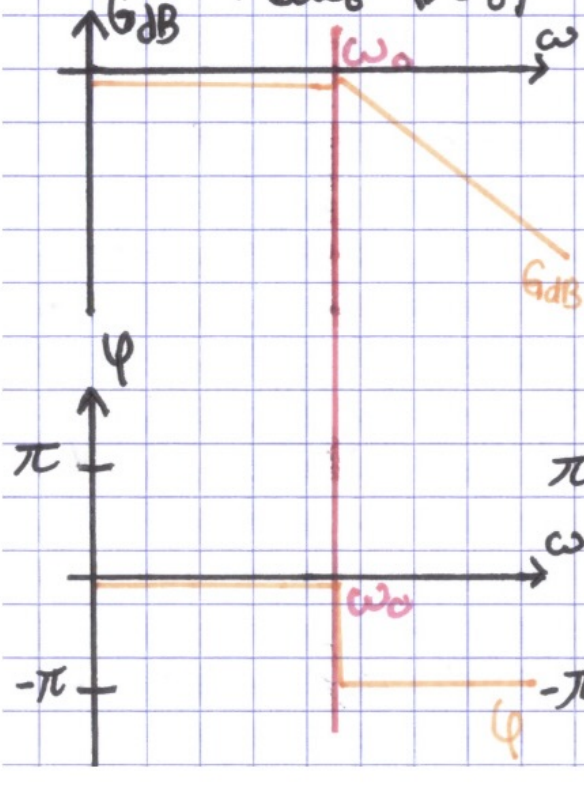
$$\underline{H} = \frac{j\frac{\omega}{\omega_0}}{1 + j\frac{\omega}{\omega_0}}$$



SECOND ORDRE

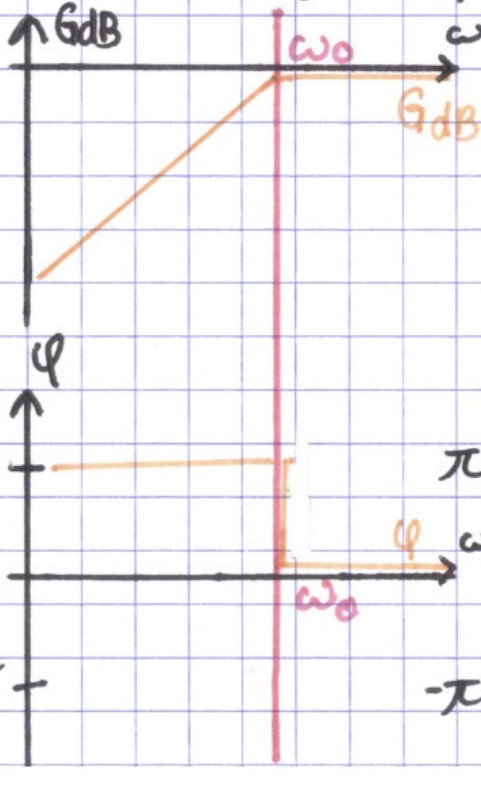
Passe bas

$$\underline{H} = \frac{1}{1 + j\frac{\omega}{Q\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2}$$



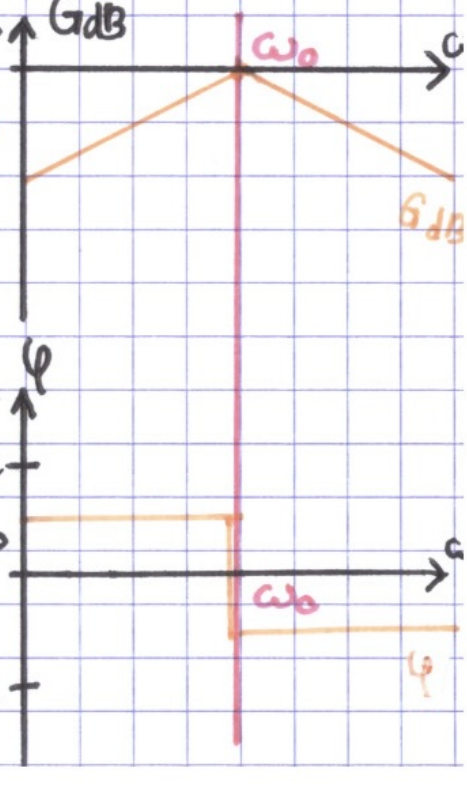
Passe haut

$$\underline{H} = \frac{\left(j\frac{\omega}{\omega_0}\right)^2}{1 + j\frac{\omega}{Q\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2}$$



Passe-bande

$$\underline{H} = \frac{j\frac{\omega}{Q\omega_0}}{1 + j\frac{\omega}{Q\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2}$$



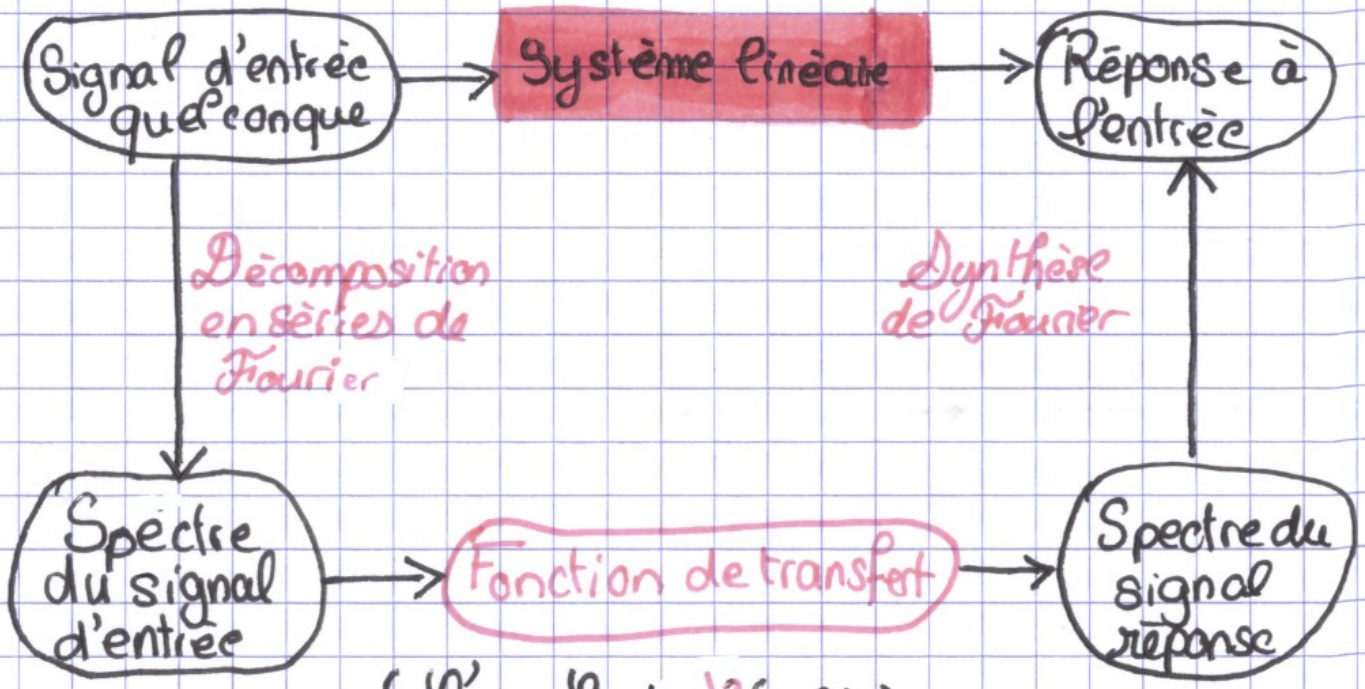
Physique Traitement du signal 2

Décomposition en séries de Fourier

$$v(t) = \langle v \rangle + \sum_{n=1}^{\infty} \underbrace{c_n \cdot \cos(n\omega_e + \varphi_n)}_{\text{harmonique de rang } n.}$$

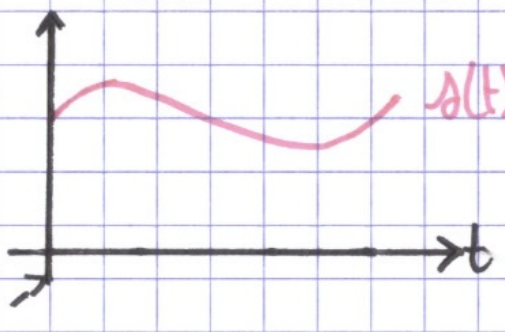
où $\omega_e = 2\pi f$, pulsation de v

- signal triangulaire : harmoniques impaires et amplitudes $\propto \frac{1}{n^2}$
- signal carré : harmoniques impaires et amplitudes $\propto 1/n$.



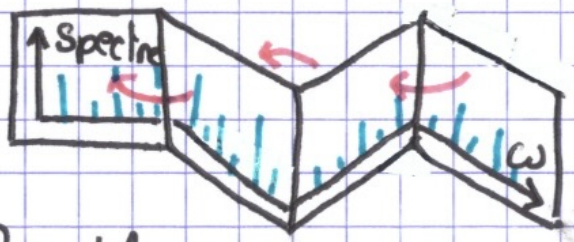
$$\begin{cases} \varphi'_n = \varphi_n + \varphi(n\omega_e) \\ S_n = G(n\omega_e) \cdot E_n \end{cases}$$

Numérisation



↓ Échantillonnage ⇝ "peigne" de Dirac

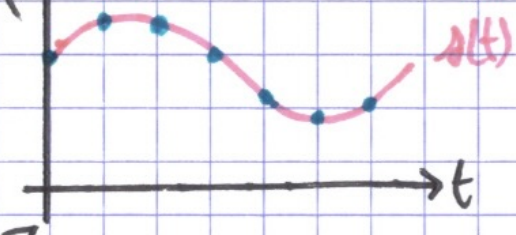
⚠ phénomène de repliement



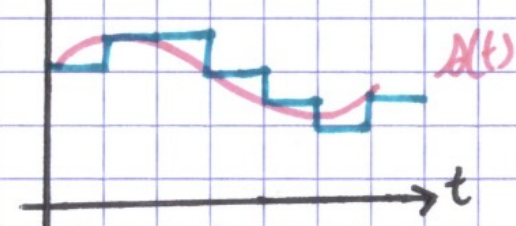
Critère de Nyquist-Shannon

$f_{éch} < 2 f_{max}$

(en théorie) filtrage



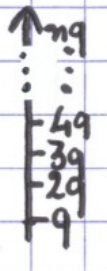
(perte d'informations) bloqueur



↓ Quantification

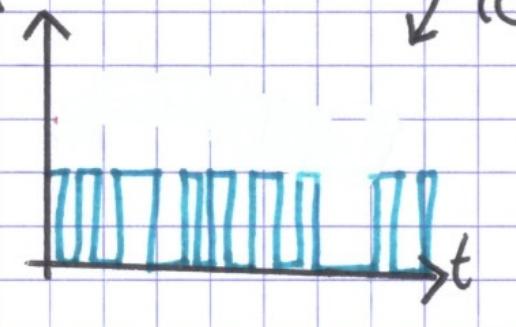
$\text{Quantum } q = \frac{PE}{2^n}$

Pas de quantification



(CNA)

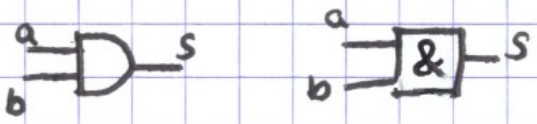
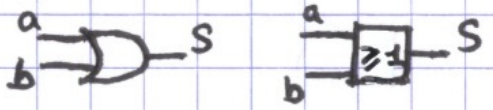
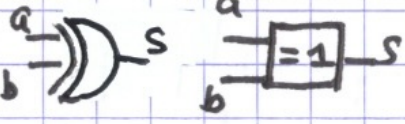
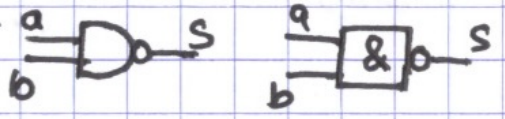
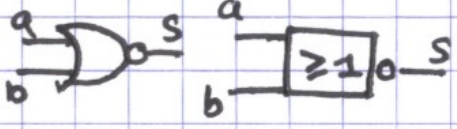
↓ Codage (en binaire) (CAN)



$$N \text{ octets} = f_{éch} \times \frac{\#\text{ bits}}{8} \times \#\text{ voies}$$

Physique Traitement du signal 3.

Portes

- **non** $S = \bar{e}$ 
- **et** $S = a \cdot b$ 
- **ou** $S = a + b$ 
- **ou exclusif** $S = a \oplus b$ 
- **non-et** $S = \overline{a \cdot b}$ 
- **non-ou** $S = \overline{a + b}$ 

portes universelles

Propriétés

▷ $a+b = b+a$, $a \cdot b = b \cdot a$ → commutativité

▷ $a+(b+c) = (a+b)+c$, $a(bc) = (a \cdot b) \cdot c$ → associativité

▷ $a \cdot (b+c) = a \cdot b + a \cdot c$, $a + (b \cdot c) = (a+b) \cdot (a \cdot c)$ → distributivité

▷ $\overline{a+b} = \bar{a} \cdot \bar{b}$, $\overline{a \cdot b} = \bar{a} + \bar{b}$ → De Morgan.